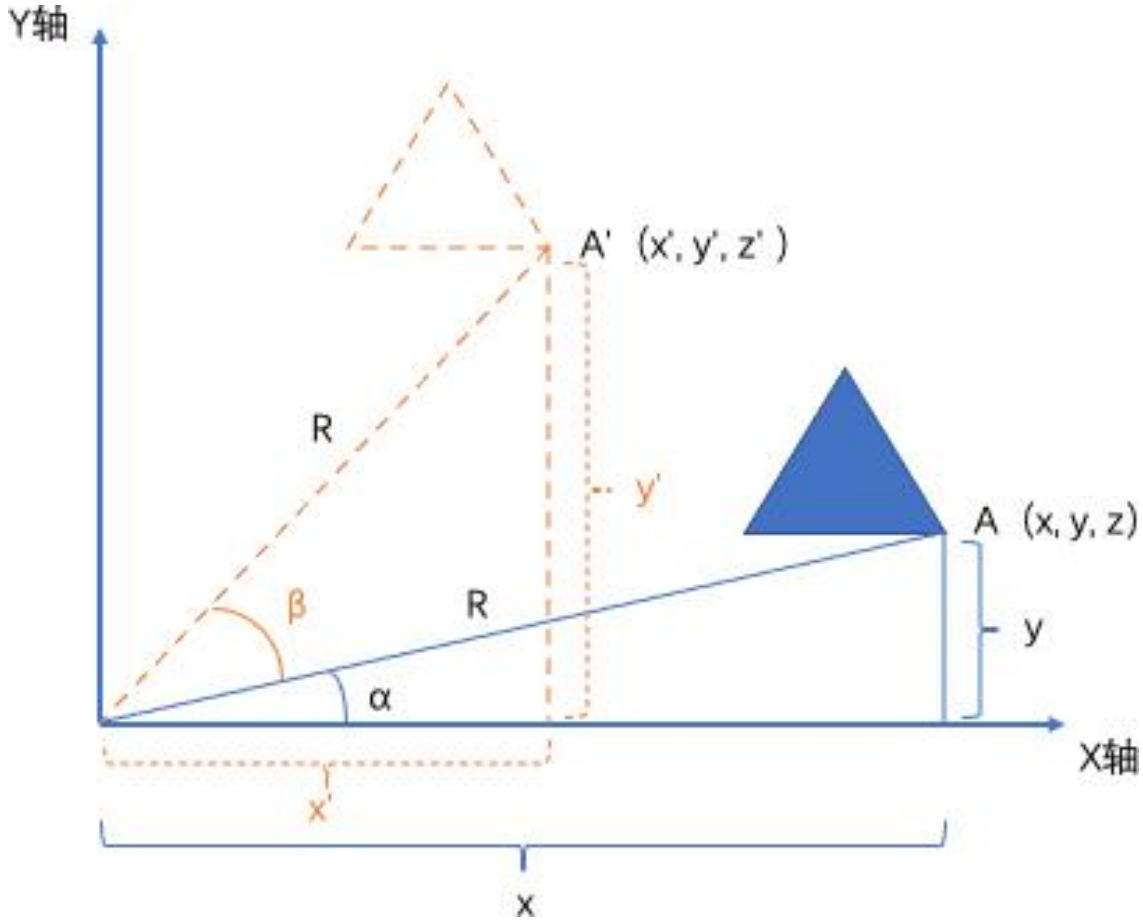
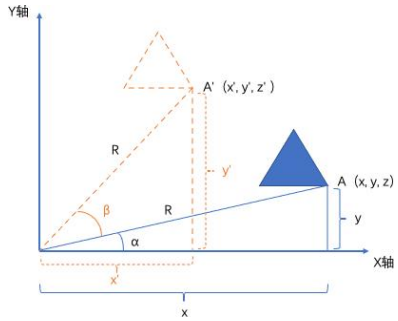
图形旋转 – 旋转矩阵


Y轴
A' (x', y', z')
R
y'
A (x, y, z)
β
R
y
α
X轴
x'
x

1. 矩阵推导

得到矩阵公式



- 顶点A

$$x = R * \cos(\alpha)$$

$$y = R * \sin(\alpha)$$

$$z = 0$$

- 顶点A'

$$x' = R * \cos(\alpha + \beta)$$

$$= R * (\cos(\alpha) * \cos(\beta) - \sin(\alpha) * \sin(\beta))$$

$$= R * \cos(\alpha) * \cos(\beta) - R * \sin(\alpha) * \sin(\beta)$$

$$y' = R * \sin(\alpha + \beta)$$

$$= R * (\sin(\alpha) * \cos(\beta) + \cos(\alpha) * \sin(\beta))$$

$$= R * \sin(\alpha) * \cos(\beta) + R * \cos(\alpha) * \sin(\beta)$$

$$z' = z$$

- 将 **顶点A** 的公式代入到 **顶点A'**

$$x' = x * \cos(\beta) - y * \sin(\beta) \quad y'$$

$$= y * \cos(\beta) + x * \sin(\beta) \quad z' =$$

$$z$$

1. 矩阵推导

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} * \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix}$$

- $ax + by + cz + d = x'$
- $ex + fy + gz + h = y'$
- $ix + jy + kz + l = z'$
- $mx + ny + oz + p = w'$

1. 矩阵推导

- ax + by + cz + w = x * cos(β) - y * sin(β)：只有当 a = cos(β)，b = -sin(β), c = w = 0 的时候，等式左右两边成立

- ex + fy + gz + h = y * cos(β) + x * sin(β)：只有当 e = sin(β) f = cos(β), g = h = 0 的时候，等式左右两边成立

1. 矩阵推导

- ix + jy + kz + l = z'：只有当 k = 1, i = j = k = 0 的时候，等式左右两边成立

- mx + ny + oz + p = 1'：只有当 m = n = o = 0, p = 1 的时候，等式左右两边成立

1. 矩阵推导

$$\begin{pmatrix} \cos(\beta), & \sin(\beta), & 0, & 0, \\ -\sin(\beta), & \cos(\beta), & 0, & 0, \\ 0, & 0, & 1, & 0, \\ 0, & 0, & 0, & 1, \end{pmatrix}$$